Quiz I, MTH 205, Fall 2019

Fayne Dsouza

Ayman Badawi

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QUESTION 1. (i) $\ell^{-1}\{\frac{1}{(s+3)^4}\} = \frac{1}{3!} \times t^3 e^{-3t} = \frac{1}{6} t^3 e^{-3t}$

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(ii) $\ell^{-1}\{\frac{2}{(s+2)^2+9}\} \Rightarrow 2\ell^{-1}\{\frac{1}{(s+2)^2+9}\} = \frac{2\sin 3t \times e^{-2t}}{3}$

$= \frac{2}{3} \sin 3t \cdot e^{-2t}$

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iii) $\ell^{-1}\{\frac{s}{3s^2+15}\} = \ell^{-1}\{\frac{s}{3(s^2+5)}\} = \frac{1}{3}\ell^{-1}\{\frac{s}{s^2+(\sqrt{5})^2}\}$

$= \frac{1}{3}\cos\sqrt{5}t$

4/4

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QUESTION 2. Use Laplace and find $y(t)$, where $y'' - 3y' - 4y = 5$, $y(0) = 0$, $y'(0) = 0$.

$\ell\{y''-3y'-4y\} = \ell\{5\} \Rightarrow \ell\{y''\} - \ell\{3y'\} - \ell\{4y\} = \ell\{5\}$

$a_2(t)=1;\ a_1(t)=3;\ a_0(t)=4;\ k(t)=5$

$\Rightarrow s^2Y(s) - sy(0) - y'(0) - 3[sY(s) - y(0)] - 4Y(s) = \frac{5}{s}$

$s^2Y(s) - sy(0) - y'(0) - 3sY(s) + 3y(0) - 4Y(s) = \frac{5}{s}$

$Y(s)[s^2 - 3s - 4] - sy(0) - y'(0) + 3y(0) = \frac{5}{s} \Rightarrow Y(s)[s^2-3s-4] - 0 - 0 + 0 = \frac{5}{s}$

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$Y(s) = \frac{5}{s(s^2-3s-4)} = \frac{5}{s(s^2-4s+s-4)} = \frac{5}{s[s(s-4)+1(s-4)]} = \frac{5}{s(s+1)(s-4)}$

$\frac{5}{s(s+1)(s-4)} = \frac{A}{s} + \frac{B}{(s+1)} + \frac{C}{(s-4)}$

$A = \frac{5}{-4} = \frac{-5}{4};\ B = \frac{5}{(-1)(-5)} = 1;\ C = \frac{5}{(4)(5)} = \frac{1}{4}$

$\Rightarrow Y(s) = \frac{-5}{4} \times \frac{1}{s} + \frac{1}{s+1} + \frac{1}{4} \times \frac{1}{s-4}$

$\ell^{-1}\{Y(s)\} = \frac{-5}{4} + e^{-t} + \frac{1}{4}e^{4t}$

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Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Quiz 2, MTH 205, Fall 2019

Ayman Badawi

$\frac{20}{20}$

QUESTION 1. (i) $\ell^{-1}\{\frac{e^{-s}}{(s-4)^2}\}$

$\ell^{-1}\left\{\frac{1}{(s-4)^2}\right\} = e^{4t}\cdot t$

$$\boxed{\ell^{-1}\left\{\frac{e^{-s}}{(s-4)^2}\right\} = \left(e^{4(t-1)}(t-1)\right)U(t-1)}$$

$\ell^{-1}\left\{\frac{1}{(s-4)^2}\right\} = t e^{4t}$

U_1

3/3

(ii) $\ell^{-1}\{\frac{e^{-s}+2}{s^2+4}\}$ $\quad \ell^{-1}\left\{\frac{e^{-s}}{s^2+4}\right\} + \ell^{-1}\left\{\frac{2}{s^2+4}\right\}$

$$= \boxed{\frac{1}{2}\sin(2(t-1))\,U(t-1) + \sin 2t}$$

3/3

QUESTION 2. Solve for $y(t)$, $y'' + 6y' + 13y = 0$, where $y(0) = 0$, $y'(0) = 2$.

$\Rightarrow s^2Y(s) - s\cancel{y(0)}^{0} - y'(0) + 6sY(s) - \cancel{6y(0)}^{0} + 13Y(s) = 0$

$\Rightarrow Y(s)(s^2+6s+13) - 2 = 0$

$\Rightarrow Y(s) = \frac{2}{s^2+6s+13}$

3/3

$\Rightarrow \ell^{-1}\left\{\frac{2}{s^2+6s+13}\right\} = y(t)$

$\Rightarrow \ell^{-1}\left\{\frac{2}{s^2+6s+9-9+13}\right\} = y(t)$

$\Rightarrow \ell^{-1}\left\{\frac{2}{(s+3)^2+4}\right\} = y(t)$

$\Rightarrow \boxed{y(t) = e^{-3t}\sin 2t}$

4/3

$\boxed{K=2}$

QUESTION 3. Solve for $y(t)$, $y'' - 4y' + 4y = tU_3(t)$, where $y(0) = y'(0) = 0$

$(t-3)U_3(t)$

$\boxed{(s-2)^2 = s^2 - 4s + 4}$

$$s^2Y(s) - \cancel{sy(0)} - \cancel{y'(0)} - 4sY(s) - \cancel{4y(0)} + 4Y(s) = \frac{e^{-3s}}{s^2}$$

$\boxed{\ell\{(t-3)U(t-3)\} = \frac{1}{s^2}e^{-3s}}$

$$\to (s^2 - 4s + 4)Y(s) = \frac{e^{-3s}}{s^2}$$

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$$\to Y(s) = \frac{e^{-3s}}{s^2(s-2)^2}$$

Partial fraction

$$\frac{A}{s} + \frac{B}{s^2} + \frac{C}{(s-2)} + \frac{D}{(s-2)^2}$$

$\boxed{B = 1/4}$, $\boxed{D = 1/4}$

$$\to y(t) = \ell^{-1}\left\{\frac{e^{-3s}}{s^2(s-2)^2}\right\}$$

$$\to y(t) = \ell^{-1}\left\{\frac{1}{s^2(s-2)^2}\right\}$$

3

$$As(s^2 - 4s + 4) + B(s^2 - 4s + 4) + Cs^2(s-2) + Ds^2$$

$$\to = \ell^{-1}\left\{\frac{1/4}{s} + \frac{1/4}{s^2} - \frac{1/4}{(s-2)} + \frac{1/4}{(s-2)^2}\right\}$$

$$As^3 - 4As^2 + 4As + Bs^2 - 4Bs + 4B + Cs^3 - 2Cs^2 + Ds^2$$

$$= 1/4 + 1/4(t-3) - 1/4e^{2(t-3)} + 1/4(t-3)e^{2(t-3)}$$

$$\boxed{y(t) = U(t-3)\left[1/4 + 1/4(t-3) - 1/4e^{2(t-3)} + 1/4e^{2(t-3)}(t-3)\right]}$$

$\boxed{A + C = 0}$ (1)

$\boxed{-4A + B - 2C + D = 0}$ (2)

$\boxed{4A - 4B = 0}$ (3)

$\boxed{4B = 1}$ (4)

$\boxed{A = 1/4}$, $\boxed{C = -1/4}$

QUESTION 4. Let $f(t) = \begin{cases} 3 & if\ 2 \le t < 5 \\ 0 & if\ t \ge 5 \end{cases}$. Find $\ell\{f(t)\}$

$$f(t) = (U_2(t) - U_5(t))3 + \cancel{[U_5(t)]0}$$

$$f(t) = 3U_2 - 3U_5$$

$$\ell\{f(t)\} = \frac{3}{s}e^{-2s} - \frac{3}{s}e^{-5s}$$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

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$$\boxed{\ell\{f(t)\} = \frac{3}{s}(e^{-2s} - e^{-5s})}$$

Name [redacted], ID [redacted]

Quiz 3, MTH 205 , Fall 2019

Ayman Badawi

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QUESTION 1. Find $x(t), y(t)$ such that $x(0) = 3, y(0) = 0$ and

$$x'(t) + x(t) - 9y(t) = 0$$
$$y'(t) + x(t) + y(t) = 0$$

$$sX(s) - \cancel{x(0)}^{3} + X(s) - 9Y(s) = 0$$
$$sY(s) - \cancel{y(0)}_{0} + X(s) + Y(s) = 0$$

① $X(s)\,(s+1) - 9Y(s) = 3$

② $X(s) + (s+1)\,Y(s) = 0$

$$X(s) = \frac{\begin{vmatrix} 3 & -9 \\ 0 & s+1 \end{vmatrix}}{\begin{array}{cc} X(s) & Y(s) \\ \end{array}\begin{vmatrix} s+1 & -9 \\ 1 & s+1 \end{vmatrix}} = \frac{3(s+1) - 0}{(s+1)(s+1) + 9} = \frac{3(s+1)}{s^2+2s+10} = \frac{3(s+1)}{(s+1)^2+9}$$

$s^2 + s + s + 1$

$\cancel{X(s) = \frac{3s + 3}{s^2+2s+10}}$

$$\mathcal{L}^{-1}\left\{X(s) = \frac{3(s+1)}{(s+1)^2+9}\right.$$

$$\boxed{x(t) = 3e^{-t}\cos(3t)}$$

$$Y(s) = \frac{\begin{vmatrix} s+1 & 3 \\ 1 & 0 \end{vmatrix}}{\begin{array}{cc} X(s) & Y(s) \end{array}\begin{vmatrix} s+1 & -9 \\ 1 & s+1 \end{vmatrix}} = \frac{0 - 3}{(s+1)(s+1)+9} = \frac{-3}{(s+1)^2+9}$$

$$\boxed{y(t) = -e^{-t}\sin(3t)}$$

10/10

10/10

QUESTION 2. Find $y(t)$, where $y^{(2)} - 4y(t) = 4U_4(t)sin(2t - 8)$, $y(0) = y'(0) = 0$ (note $U_4(t) = U(t-4)$))

$$s^2Y(s) - \cancel{sy(0)} - \cancel{y'(0)} - 4Y(s) = \frac{8e^{-4s}}{s^2+4}$$

$$\mathcal{L}\{4U_4 \sin(2t-8)\}$$

$$4\left[e^{-4s}\mathcal{L}\{\sin(2(t+4)-8\right.$$

$$4\left[e^{-4s}\mathcal{L}\{\sin(2t)\}\right]$$

$$4\left[e^{-4s}\frac{2}{s^2+4}\right]$$

$$Y(s)(s^2-4) = \frac{8e^{-4s}}{s^2+4}$$

$$Y(s) = 8e^{-4s}\overbrace{\frac{1}{(s^2-4)(s^2+4)}}^{F(s)} \quad 8\, f(t-4)\, U_4$$

$$F(s) = \frac{1}{(s^2-4)(s^2+4)}$$

$$\cancel{s^2+4} - s^2+4$$

$$\frac{1}{s^2-4} - \frac{1}{s^2+4} = \frac{8}{s^2-4}$$

$$\mathcal{L}^{-1}F(s) = \mathcal{L}^{-1}\frac{1}{8}\left[\frac{1}{s^2-4} - \frac{1}{s^2+4}\right]$$

$$f(t) \cancel{F(s)} = \frac{1}{8*2}\sinh(2t) - \frac{1}{8*2}\sin(2t) = \frac{1}{16}\left[\sinh(2t) - \sin(2t)\right]$$

$$y(t) = \frac{8}{16}\left[\sinh(2(t-4)) - \sin(2(t-4))\right]U_4$$

$$= \frac{1}{2}\left[\sinh(2(t-4)) - \sin(2(t-4))\right]U_4$$

✓ 6/6

QUESTION 3. Find $y(t)$, where $y' - 2y(t) = 2^t$, $y(0) = 0$

$$e^{t\ln 2} \quad \mathcal{L}\{e^{\ln 2\, t}\} = \frac{1}{s-\ln 2}$$

$$sY(s) - \cancel{y(0)} - 2Y(s) = \frac{1}{s-\ln 2}$$

$$\cancel{s-2} \; \cancel{s} + \ln 2$$

$$\cancel{sY}\; Y(s)(s-2) = \frac{1}{s-\ln 2}$$

$$\frac{1}{s-\ln 2} - \frac{1}{s-2} = \frac{-2+\ln 2}{}$$

$$Y(s) = \frac{1}{(s-\ln 2)(s-2)}$$

$$\mathcal{L}^{-1}Y(s) = \mathcal{L}^{-1}\frac{1}{-2+\ln 2}\left[\frac{1}{s-\ln 2} - \frac{1}{s-2}\right] = \frac{1}{-2+\ln 2}\left[e^{\ln 2 t} - e^{2t}\right]$$

4/4

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name ______________, ID ______

Quiz IV , MTH 205 , Fall 2019

Ayman Badawi

20/20

QUESTION 1. Find the general solution for $y(t)$ if $y^{(5)} + 8y^{(4)} + 7y^{(3)} = 0$.

$m^5 + 8m^4 + 7m^3 = 0$

$m^3(m^2 + 8m + 7) = 0$

$m = 0$, $m = -1$, $m = -7$
thrice

$c_1 e^{0t} + c_2 t e^{0t} + c_3 t^2 e^{0t} + c_4 e^{-t} + c_5 e^{-7t}$

$y(t) = c_1 + c_2 t + c_3 t^2 + c_4 e^{-t} + c_5 e^{-7t}$

7/7

QUESTION 2. Find the general solution for $y(t)$ if $y^{(2)} - 8y' + 16y = 0$

$m^2 - 8m + 16 = 0$

$m = 4$
twice

$y(t) = c_1 e^{4t} + c_2 t e^{4t}$

6/6

QUESTION 3. Find the general solution for $y(t)$ if $y^{(2)} + 8y' + 25y = 0$

$m^2 + 8m + 25 = 0$

$(m+4)^2 - 16 + 25 = 0$

$(m+4)^2 = -9$

$m = -4 \pm 3i$

$y = e^{-4t}\left[c_1 \cos(3t) + c_2 \sin(3t)\right]$

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Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: abadawi@aus.edu, www.ayman-badawi.com

Name————————————————, ID ————————

Quiz 5, MTH 205, Fall 2019

Ayman Badawi

Total = $\overline{\quad\quad 15}$

QUESTION 1. Solve for $y(x)$.
$y' + 3x^2y = e^x(3x^2 + 1)$

QUESTION 2. Solve for $y(x)$
$xy' + y = x^2e^xy^{13}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`

Quiz 7, MTH 205, Fall 2019

Ayman Badawi

$\frac{20}{20}$

Total = $\frac{\quad}{20}$

QUESTION 1. Solve
$dy/dx = e^{3x+4y}$

$$\frac{dy}{dx} = e^{3x} \cdot e^{4y}$$

$$\int \frac{1}{e^{4y}}\, dy = \int e^{3x}\, dx$$

$$\int e^{-4y}\, dy = \frac{e^{3x}}{3} + C$$

$$\boxed{-\frac{e^{-4y}}{4} = \frac{e^{3x}}{3} + C}$$

$$\frac{-1}{4} e^{-4y} = \frac{e^{3x}}{3} + C$$

$$e^{-4y} = -\frac{4}{3} e^{3x} - 4C$$

$$-4y = \ln\left|\frac{-4}{3} e^{3x} - 4C\right|$$

$$y = \frac{-\ln\left|\frac{-3}{4} e^{3x} - 4x\right|}{4}$$

$\frac{6}{6}$

QUESTION 2. $x(1+y^2)^{0.5}\, dx = y(1+x^2)^{0.5}\, dy$

$$\int \frac{x}{\sqrt{1+x^2}}\, dx = \int \frac{y}{\sqrt{1+y^2}}\, dy$$

$$\boxed{\sqrt{(1+x^2)} = \sqrt{(1+y^2)} + C}$$

$$\int \frac{y}{\sqrt{1+y^2}}\, dy$$

$u = 1+y^2$

$du = 2y\, dy$ $\quad \frac{1}{2}\int \frac{du}{\sqrt{u}}$

$$\frac{1}{2}\int u^{-1/2}\, du = \frac{1}{2} \frac{u^{1/2}}{1/2}$$

$$= u^{1/2} = (1+y^2)^{1/2}$$

$\frac{6}{6}$

QUESTION 3. $(e^{2y} - ycos(xy))\,dx + (2xe^{2y} - xcos(xy) + 2y)\,dy = 0$

F_x $\quad$ F_y

$F_{xy} = 2e^{2y} + yx\sin(xy) - \cos(xy)$

$F_{yx} = 2e^{2y} + xy\sin(xy) - \cos(xy)$

exact

$F(x,y) = \int F_x\,dx = \int e^{2y} - y\cos(xy)\,dx$

$e^{2y}x - \dfrac{\cancel{y}\sin(xy)}{\cancel{y}} = e^{2y}x - \sin(xy) + C(y)$

$F_y = 2xe^{2y} - x\cos(xy) + C'(y) = 2xe^{2y} - x\cos(xy) + 2y$

$\int C'(y) = \int 2y$

$C(y) = \dfrac{\cancel{2}y^2}{\cancel{2}} + C$

$\boxed{\cancel{F(x,y)} = e^{2y}x - \sin(xy) + y^2 + C} = 0$

$\underbrace{y}_{u}\,\underbrace{\cos(xy)}_{v}$

$uv' + vu'$

$-yx\sin(xy) + \cos(xy)$

$-\underbrace{x}_{u}\,\underbrace{\cos(xy)}_{v}$

$uv' + vu'$

$+xy\sin(xy) \mp \cos(xy)$

$\frac{8}{8}$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu`, `www.ayman-badawi.com`

Name———————————————, ID ———————

Quiz 6, MTH 205, Fall 2019

Ayman Badawi

Total = $\overline{20}$

QUESTION 1. Solve
$dy/dx = e^{3x+4y}$

QUESTION 2. $x(1+y^2)^{0.5}\ dx = y(1+x^2)^{0.5}\ dy$

QUESTION 3. $(e^{2y} - ycos(xy))\ dx + (2xe^{2y} - xcos(xy) + 2y)\ dy = 0$

Faculty information

Ayman Badawi, Department of Mathematics & Statistics, American University of Sharjah, P.O. Box 26666, Sharjah, United Arab Emirates.
E-mail: `abadawi@aus.edu, www.ayman-badawi.com`